

Turbulent diffusion in a stratified fluid

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(Received 7 August 1962)

The behaviour of cylinders of fluid injected vertically downwards into a similar stationary fluid has been observed with particular reference to the effect of a stable density stratification in the stationary fluid upon the penetration and upon the mixing between the injected and ambient fluids.

When the ambient fluid is of uniform density, it is found that the motion is approximately self-preserving, and that the energy decay rate per unit mass of moving fluid follows the same law as does the decay rate in a turbulent fluid.

The observations demonstrate that the effect of the density stratification in the ambient fluid is principally upon the gross motion and only slightly upon the smaller scales of motion.

Estimates are given of the maximum conversion of kinetic energy to potential energy, and of the ultimate loss of energy to buoyancy, as a function of a Richardson number determined from the initial conditions of volume, velocity, density gradient, density, and the acceleration of gravity.

1. Introduction

Visual observation of turbulent fluids containing some marker for flow visualization gives the impression that much of the motion consists of 'puffs' of fluid. These seem to be created by rather rapid local accelerations and then to move with little change of direction, gradually slowing as they entrain surrounding fluid until they no longer have appreciable velocity relative to their surroundings. This impression is particularly strong in the case of flows such as wakes, jets, and boundary layers, where the turbulent fluid can be observed to encroach upon the non-turbulent surrounding fluid. We therefore consider that the motion of puffs deliberately injected into stationary fluid may not be altogether irrelevant to the general problem of the nature of turbulence—as well as being interesting in itself. In particular the behaviour of a puff introduced into a fluid with a stable density gradient may be especially instructive.

With these concepts in view, discrete volumes of fluid penetrating a still fluid in a vertically downward direction have been studied as a function of the initial volume and velocity of the penetrating fluid. The still fluid had either the same density as the injected fluid or was stably stratified. In all cases the injected fluid was of nearly the same density as the surroundings at the point of injection.

The injected and tank fluids contained chemicals which produced a finely divided precipitate when mixing occurred on a molecular scale. Observation of

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this precipitate, both as to amount and position, permitted estimates of the detailed mixing process occurring between the injected and ambient fluid, and also provided flow visualization of the progress of the injected fluid with its associated entrained fluid.

If the behaviour of isolated fluid volumes moving in quiet surroundings can be shown to have features in common with the behaviour of a turbulent fluid, inferences on the behaviour of different scales of motion may be obtained which cannot be obtained from a turbulent fluid where many scales and orientations occur simultaneously.

From the variation of velocity and scale of the moving fluid with time, the energy decay rate per unit mass is shown to obey $dw^2/dt \propto w^3/L$, where L is a length typical of the moving fluid and w the vertical velocity. The same energy decay rate per unit mass holds in a turbulent fluid (Townsend 1956). This implies that a fluid containing a large number of randomly oriented penetrating volumes would have features in common with a naturally turbulent fluid resulting from a shear flow, and that calculations made from the observations on isolated moving volumes may perhaps be applied with some confidence to a turbulent field.

To show the effects of the stable density gradient on the motion, numerical estimates have been obtained for several energy ratios as a function of a dimensionless combination of the initial conditions which resembles the reciprocal of a Richardson number and is given by

$$\frac{1}{R_i} = \frac{\rho_0 w_0^2}{g(d\rho/dz)V_0^{3/2}}$$

where ρ_0 is the density of the injected fluid, w_0 is the velocity of injection, V_0 is the volume injected, z is the vertical displacement, positive downwards, and $d\rho/dz$ is the density gradient existing in the tank. The number, $1/R_i$, varied from 37 to 1140 in the experiment; changes in the initial velocity and volume being the principal cause of variation.

The motion of discrete volumes of fluid moving under their own buoyancy into quiet surroundings has been studied by Scorer (1957, 1958), and is not considered here.

2. Method

The initial motion was caused by injecting a cylinder of fluid vertically downwards into a tank of still fluid through a nozzle projecting below the surface. To prevent the formation of vortex rings, the nozzle, which was a section of glass tubing, had a coarse screen cemented on the end. The screen served to distribute the concentrated vorticity caused by injection, and destroy the ring. Vortex rings in real fluids have been studied experimentally by Turner (1957), but were considered to be outside the scope of the present study. The motion desired here was turbulent motion with, if possible, no ordered vorticity.

To study the detailed mixing, some physically observable quantity must be introduced which alters when molecular mixing occurs between the injected and entrained fluid. The chemical reaction between aqueous solutions of sodium carbonate in the injected fluid, and calcium hydroxide in the tank fluid, appears

to fulfil the requirements quite well. Since the reaction between ions in solutions occurs very rapidly when the fluids are thoroughly mixed (Moelwyn-Hughes 1957), the rate of formation of calcium carbonate will be controlled by the rate at which physical mixing brings the reagents into intimate contact. The settling velocity of the cloud of calcium carbonate particles was sufficiently small that it could be neglected in comparison with the fluid velocities.

Quantitatively, the amount of calcium carbonate formed was determined by photoelectric measurements of the light scattered by the cloud of precipitate at right angles to the incident beam. The assumption of a linear relationship between the volume of precipitate and the photocell output was justified by the observed relation between injected volume and maximum photocell output when the injected volume was varied. The ratio, maximum output/injected volume, was constant within 10 % over the range of volumes used when reagent concentrations and light intensity were held constant.

To provide a stratification of density in the tank, sodium chloride was dissolved in the lower portion. Manual mixing served to produce an approximately linear density gradient as measured by 12 neutrally buoyant glass floats (approximately 0.5 cm diameter) which were calibrated previously.

All observations of the cloud of precipitate were made from a 16 mm Kine photographic record, taken at a nominal 16 frames per second. At the same time the output of the photocell was displayed on one beam of a dual beam oscilloscope, and, by means of a system of mirrors, was photographed simultaneously with the precipitate cloud.

Apparatus

A sketch of the apparatus is shown in figure 1. The fluid to be injected was placed in a beaker at a height of 65 cm above the surface of the tank fluid. Into this beaker dipped a siphon tube, the end of which was closed, except during injections, by a rubber hemisphere which formed the valve. A set of motor driven cams was constructed in order to give different valve opening times. The time of valve opening was determined by having the cam contact apply a small deflexion voltage to the second oscilloscope beam while the valve was open. A short tube on the top of the siphon permitted withdrawal of air from the siphon tube to facilitate filling the injection system with fluid.

For the measurement of the injected volume, a simple but effective technique was devised: a graduated section in the fall tube contained a loosely fitted polythene bead, which would slowly float up to a constriction in the tube when the valve was closed. When injection was taking place the bead fell with the fluid, thus permitting measurement of the injected volume.

An adjustable valve in the fall tube provided a means of volume regulation independent of the on-off valve. In combination with the various cams, this arrangement made possible some range of velocity-volume combinations.

The average initial velocity w_0 of the injected fluid was calculated from the valve-opening time, the volume measurement, and the area of the nozzle tube. This initial velocity was assumed to be equal to the maximum velocity attained if the acceleration were constant, that is to twice the volume divided by the

product of nozzle area and valve-opening time. The momentum thus calculated agreed reasonably well with the momentum determined from the film record, although momentum determined from the film was approximate since volume measurements from the film record were subject to considerable error.

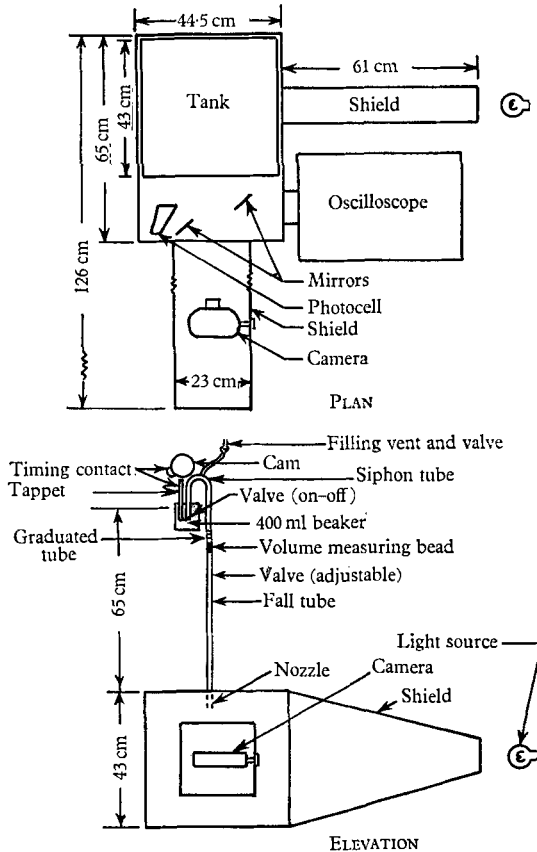


FIGURE 1. Experimental apparatus.

The nozzle was a section of glass tubing 0.540 cm in diameter, the end of which was covered with a coarse screen consisting of 0.33 cm diameter wires spaced approximately 0.1 cm apart.

The tank was of plate glass. In order to minimize the amount of stray light present, it was painted dull black except for two windows on adjacent sides. One of these admitted light from the source, the other was used for observation.

The tank fluid consisted of 1.28 g/l of $\text{Ca}(\text{OH})_2$ in distilled water. It was prepared in a carboy, filtered into the tank and protected from the atmosphere by a 0.5 cm layer of kerosene on the surface. The injected fluid contained 10 g/l of $\text{Na}_2\text{CO}_3 \cdot 10\text{H}_2\text{O}$ in distilled water. Hence in terms of the reaction to form CaCO_3 , this fluid was twice as concentrated as the tank fluid.

When a density gradient was required, the bottom portion of the tank fluid contained up to 10% NaCl. Measurement of the density gradient was accomplished by means of small glass floats approximately 0.5 cm in diameter. Twelve

of these were made with densities covering the range in the tank. They were calibrated and placed in a tall cage with a glass front and back, and with coarse plastic screen sides. The cage was set on the bottom of the tank near one of the windows so the floats could be observed from outside. The density gradients used were close to 0.003 g cm^{-4} and were very nearly constant over the region in which the motion occurred.

Illumination was provided by a 300 W incandescent lamp placed 100 cm from the centre of the tank. Over the region in which the motion took place this gave a light intensity which was constant to within 5% of its value at the centre.

Light scattered at right angles was measured by a General Electric PV3 photovoltaic cell, the output of which was amplified and applied to one of the vertical inputs of the oscilloscope.

To obtain the position of the centre of the cloud of precipitate from the film record, an enlarged image was projected on a screen. Distances in the tank were determined by photographing a graduated vertical scale immersed in the fluid at the centre of the tank and projecting this photograph on the screen so that a grid could be drawn on the screen reading directly, in actual distance in the tank.

The centre of the cloud of precipitate was estimated visually and its displacement from the tip of the nozzle recorded. To check this visual estimate, the light intensity distribution for enlarged images of four dissimilar clouds were scanned with a photocell, calibrated from a step exposure placed on each film. In each case the calculated centre agreed with the estimated centre to within 3 mm (in the tank).

All time measurements were made from the film record, using the calibrated sweep speed of the oscilloscope (0.25 sec/in. to 0.10 sec/in.) to obtain an accurate value for the camera film speed.

3. Data

Detailed mixing

The observed proportionality between the volume injected and the maximum photocell output under the same illumination and tank conditions, supports the assumption of a linear relationship between the increment of photocell output over the background output and the progress of the chemical reaction. As the reaction rate is controlled by the physical mixing, the increment in the photocell output is proportional to that volume of injected fluid which has been mixed on a molecular scale with the ambient fluid.

We define the quantity Q to be the fractional completion of the reaction, i.e. the ratio (increment in photocell output)/(maximum increment). Q is thus independent of the small changes in illumination caused by changes in the transparency of the fluid and of the tank walls. Q as a function of time t can be determined from the oscilloscope beam deflexion, recorded on the film.

Experiments were carried out with initial volumes ranging from 0.065 cm^3 to 0.8 cm^3 . For the larger volumes initial velocities of from 9.4 to 55 cm/sec were possible. For the smallest volume, however, an average initial volume of 9.4 cm³/sec was the greatest that could be obtained from the apparatus.

Up to ten repeat runs were made with each combination of initial conditions in order to obtain an average behaviour and to permit the calculation of standard deviations.

An example is shown in figure 2 for an initial volume of 0.81 cm^3 and an initial velocity of 32 cm/sec . Two curves are shown against time. The solid line represents the results when the tank fluid was of uniform density and the dashed curve when the tank fluid was stably stratified with a density gradient 0.0025 g cm^{-4} .

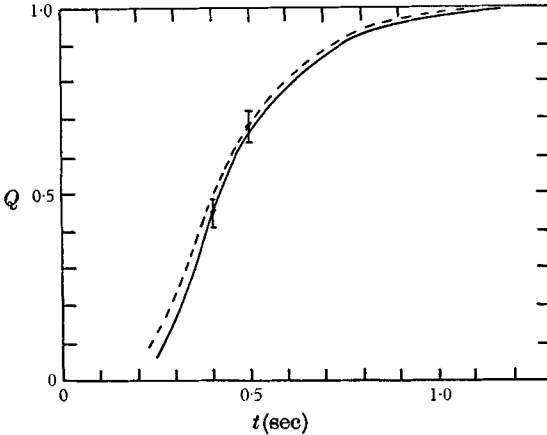


FIGURE 2

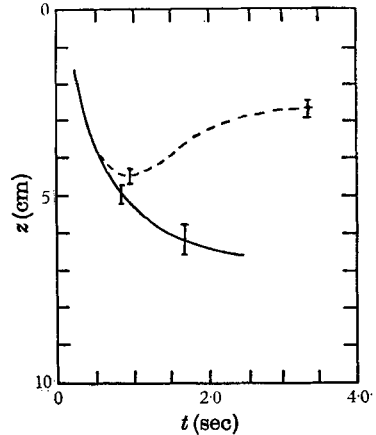


FIGURE 3

FIGURE 2. Detailed mixing. Q represents the fractional completion of the reaction and t the time after initiation. —, Mean curve for four observations in uniform fluid. ----, Mean curve for three observations in fluid with density stratification of 0.0025 g cm^{-4} . In each case the initial conditions were $V_0 = 0.82 \text{ cm}^3$ and $w_0 = 32 \text{ cm sec}^{-1}$. The vertical bars indicate ± 1 standard deviation.

FIGURE 3. Centre displacement from tip of nozzle: —, in uniform fluid; ----, in stratified fluid. The curves are computed from the same sets of experiments as figure 2.

Centre displacement

An example of the displacement z of the centre of the cloud of precipitate from the tip of the nozzle as a function of time is shown in figure 3. The example shown had the same initial conditions as figure 2 and again the results for uniform and stratified fluid are plotted on the same graph.

Horizontal spread

When the density of the tank fluid was uniform and equal to the density of the injected fluid, successive outlines of the cloud of precipitate, as it progressed, defined a cone of half angle which varied from 15° to 21° . There was no systematic dependence on the initial conditions of volume and velocity.

When the fluid in the tank was stratified, the cloud penetrated some distance, then reversed direction, and eventually came to rest. The horizontal extent of the cloud moving into stratified ambient fluid was found to be almost the same as into uniform ambient fluid when measured at the same time after initiation.

4. Interpretation of the data

Velocity and displacement in uniform surroundings

In a uniform surrounding fluid of the same density, the fact that successive outlines of the cloud of precipitate fall into a cone, suggests that the turbulent velocities inside the cloud which cause the expansion are proportional to the velocity of the centre of the cloud.

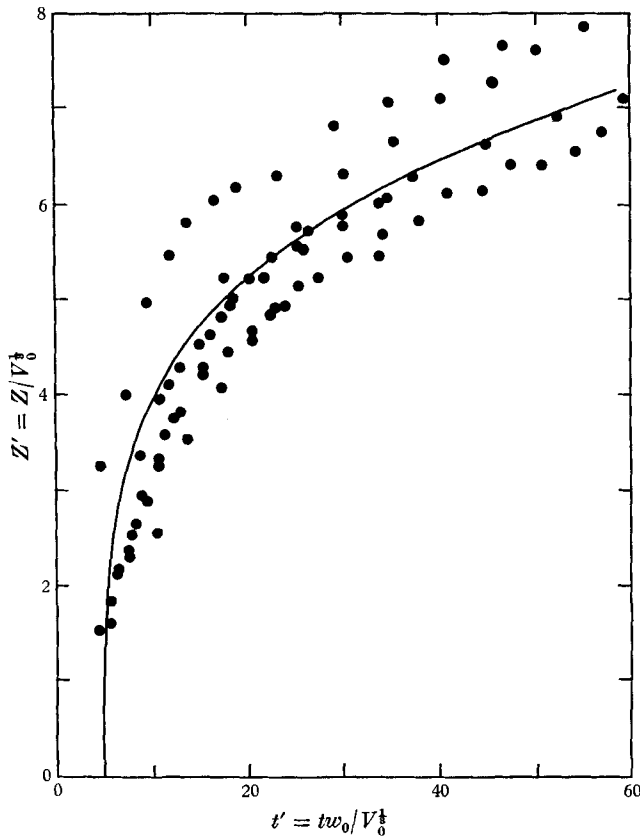


FIGURE 4. Similarity in neutrally buoyant surroundings. Z is the displacement of centre of cloud from nozzle tip (cm); V_0 , initial volume (cm^3); t , time after initiation (sec); and w_0 , initial velocity (cm sec^{-1}).

If all velocities remain proportional to one another, the flow is of the type called self-preserving, and is independent of the Reynolds number if this is sufficiently large. For a self-preserving flow, the form of the self-preserving functions are universal for any one type of flow (Townsend 1956). It follows that a non-dimensional plot of any two variables should show the same form for all flows of a given type. Such a plot is shown in figure 4, where $Z/V_0^{1/3}$ is plotted against $tw_0/V_0^{1/3}$ for all the data obtained. Z is the displacement from the tip of the nozzle, t is the time after initiation, and w_0 and V_0 represent initial velocity and volume as before. The considerable scatter is attributed to the different intervals of time and distance required to establish self-preservation. In addition, the values V_0

and w_0 separately relate to initial values and not necessarily to the flow after it has become self-preserving.

Assuming self-preservation, a dimensional argument yields an expression for displacement and velocity as a function of time, if virtual origins are chosen such that $z = 0, w = \infty$ at $t = 0$. Since the density difference between the moving and ambient fluid is zero, no buoyant forces exist, and the vertical momentum of

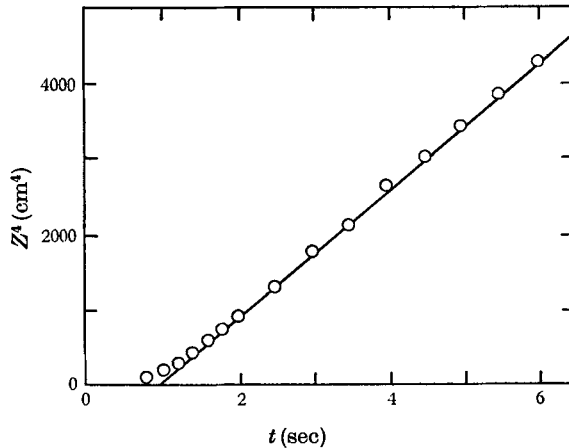


FIGURE 5. Comparison of experimental observations (points) with the form of the self-preserving solution (line) for the depth of penetration of a puff as a function of time. $V_0 = 0.80 \text{ cm}^3$; $w_0 = 9.4 \text{ cm/sec}$.

the moving fluid, which includes the injected and entrained fluid, remains constant. That is, $Vw = M$, the constant momentum of the cloud. Dimensionally we must have, for self-preservation,

$$\frac{dV}{dt} = -\frac{V}{w} \frac{dw}{dt} \propto V^{\frac{3}{2}} w, \quad (1)$$

whence

$$V \propto t^{\frac{3}{2}}, \quad w \propto t^{-\frac{3}{2}}.$$

Our observations are of the displacement Z , so to effect a comparison we integrate the second of these to obtain $Z \propto t^{\frac{1}{2}}$.

The line shown on figure 4 is of this nature: $Z'^{\frac{1}{2}} = 50(t' - 5.0)$, where

$$Z' = Z/V_0^{\frac{1}{2}}, \quad t' = tw_0 V_0^{-\frac{1}{2}}$$

and the constants have been chosen empirically. Clearly it corresponds well to the trend of the data. No correction has been made for the displacement of the virtual origin from the tip of the nozzle since this displacement varies from one set of initial conditions to the other. No systematic relation between the displacement of the virtual origin and the initial conditions has been found.

For an individual run, where an optimal virtual origin may be chosen, the fit with the self-preserving form is much better, as is shown in figure 5.

As was pointed out in the introduction, this self-preserving solution provides the same rate of energy decay per unit mass as is found for homogeneous turbulent fields, namely $dE/dt \propto E^{\frac{3}{2}}/L$, where E is the energy density and L a characteristic

scale. For with self-preservation the energy density is proportional to w^2 , and L to $V^{\frac{1}{2}}$, so that from equations (1) we see that

$$dw^2/dt \propto t^{-\frac{1}{2}} \quad \text{and} \quad w^3/V^{\frac{1}{2}} \propto t^{-\frac{3}{2}}/t^{\frac{1}{2}} = t^{-2}.$$

Motion in uniformly stratified surroundings

Where the surrounding fluid has a density which increases with depth, the moving fluid experiences buoyant forces which depend upon the entrainment rate.

The additional notation used in this section is given below:

- ρ_0 mean density of the moving fluid at the point of injection $Z = 0$;
- ρ_{z0} density of the surroundings at $Z = 0$;
- ρ_m mean density of the moving fluid at the point of maximum penetration Z_m ;
- ρ_{zm} density of surroundings at Z_m ;
- $\rho_{eq} = \rho_{zeq}$ density of the moving fluid where it finally comes to rest.

If $\rho_0 = \rho_{z0}$ the descending fluid always finds itself surrounded by heavier fluid. As the volume increases by entrainment of surrounding more dense fluid, the average density increases. The kinetic energy decreases from losses due to entrainment and from work done against the buoyant forces. The potential energy of the moving fluid, zero at the start of the motion, increases since it is necessarily positive at Z_m . After coming to rest the fluid rises under buoyant forces, usually overshoots Z_{eq} and eventually comes to rest at Z_{eq} .

We may obtain a lower limit for the potential energy of the displaced fluid at the points of maximum penetration Z_m by making the following assumptions. (a) The density of the displaced fluid at Z_m is equal to the density of the surroundings at Z_{eq} where the motion finally ceases. This must give a maximum value for the density at Z_m since the greater part of the ensuing path is through fluid of density greater than ρ_{zeq} . (b) The volume of the displaced fluid at Z_m is determined from mixing of the initial volume V_0 at density ρ_0 with fluid of the maximum density encountered, that is ρ_{zm} . This yields the smallest volume and therefore the smallest potential energy consistent with assumption (a).

The values of this potential energy have been divided by the initial (kinetic) energy of the injected fluid to form a dimensionless ratio which is plotted in figure 6 as a function of a dimensionless number computed from the initial conditions ρ_0 , w_0 , g , $d\rho/dz$ and V_0 . A suitable dimensionless combination is

$$\frac{\rho_0 w_0^2}{g(d\rho/dz) V_0^{\frac{3}{2}}}.$$

This dimensionless form resembles the reciprocal of a Richardson number, differing from it by the replacement of $(du/dz)^2$ by $w_0^2/V_0^{\frac{3}{2}}$, and is referred to as $1/R_i$.

The results shown in figure 6 indicate that as much as 20 % of the initial energy appears as potential energy for small values of $1/R_i$. The ratio decreases rapidly and approaches zero for $1/R_i$ approximately equal to 1000.

Although we have calculated a lower limit for the potential energy, the actual amount cannot be much greater. As can be seen from figure 3, for a large portion of the time the moving fluid is in the neighbourhood of Z_m , so a large proportion

of the entrainment should occur here. It is unlikely that the lower limit is an underestimate by as much as a factor of two.

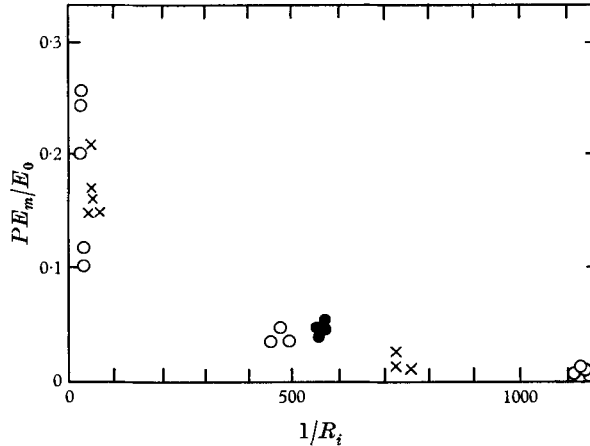


FIGURE 6. Ratio of potential energy at maximum penetration to initial energy (PE_m/E_0), as a function of $1/R_i$. Initial volumes $\circ = 0.8 \text{ cm}^3$, $\bullet = 0.6 \text{ cm}^3$, $\times = 0.4 \text{ cm}^3$.

Effect upon the surrounding fluid

Using the assumption of mixing only at the depth of maximum penetration Z_m , it is possible to calculate from Z_m , Z_{eq} , and the initial conditions, the fraction of the initial energy which is utilized in breaking down the density stratification. This is performed by calculating the change in potential energy which results if a volume of fluid at Z_0 is transported to Z_m , mixed with the requisite amount of fluid of density ρ_m , and the resulting mixture deposited at Z_{eq} . It should be noted that in this case our assumptions lead not to a lower limit, but to an estimate which may be either somewhat too high or somewhat too low.

The results are shown in figure 7 as a function of $1/R_i$. At low values of $1/R_i$, where the penetrations are small and relative errors in measurements are proportionately large, the points are badly scattered. At higher values of $1/R_i$ the points are well grouped near to a value of 0.03. As a representative figure, we could say that approximately 3% of the initial energy is used to alter the density stratification over the range of $1/R_i$ covered in the experiment.

Comparison of neutrally buoyant and stable surroundings

If the horizontal extent of the moving cloud is plotted as a function of time, the curves obtained for uniform and stably stratified surroundings fall very closely together for the same initial conditions of volume and velocity. This is true as long as the active motion persists. After the cloud, in stably stratified surroundings, comes to rest at its equilibrium position, it spreads slowly under the influence of gravity, indicating that its density is relatively uniform throughout.

A comparison has been made between the potential energy of the displaced fluid in stably stratified surroundings, and the kinetic energy of the moving fluid in neutral surroundings with the same initial conditions of volume and velocity

and at the same time after initiation. The ratio, potential energy at maximum penetration divided by kinetic energy at the same time in neutral surroundings has a mean value of 0.8 over the range of $1/R_i$ from 30 to 500. For values of $1/R_i$ equal to 1100 the energy ratio fell to zero. The calculations of the potential energy depended upon the observation of the rise in the displaced fluid from its point of maximum penetration. For large values of $1/R_i$ the cloud dissipated rapidly and it is possible that a slight unobserved rise occurred, so it is possible that the small values at $1/R_i$ equal to 1100 are in error. Since the calculations of the potential energy yielded a minimum value it is not improbable that the energy ratio does not differ greatly from 1 over a large range of initial conditions.

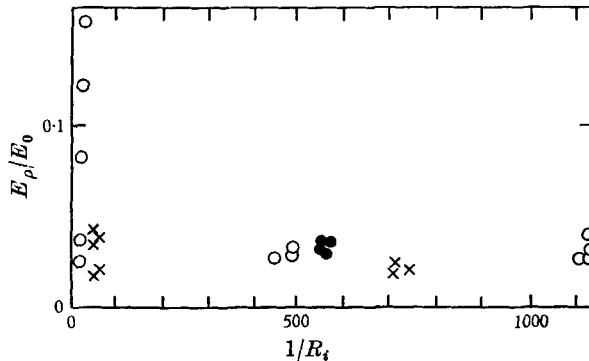


FIGURE 7. Ratio of energy lost to the density stratification, to initial energy as a function of $1/R_i$. The symbols have the same meaning as in figure 6.

Detailed mixing

The mixing between the injected and ambient fluids can be regarded as a two-stage process. In the first stage the large eddies of the turbulent fluid entrain ambient fluid and incorporate it into the turbulent region. The small eddies, which account for almost all the velocity gradient although little of the energy of the flow, then reduce the scale of the resulting inhomogeneities to the extent that molecular diffusion is able to smooth them out. The phenomenon is thought to be rather similar to the well known transfer of turbulent energy down the spectrum from small to large scales and ultimately to dissipation by viscosity (see, for example, Batchelor 1959).

The measurements reported herein give some insight into the relative rates of the two stages. Momentum considerations reveal the rate of gross entrainment, while the rate of increase of the quantity of precipitate is a measure of the detailed mixing. Thus if at time t the cloud velocity has dropped from its initial value w_0 to w , then the momentum equation tells us that there has been a gross entrainment of a volume

$$V_0 \left(\frac{w_0}{w} - 1 \right).$$

On the other hand, if the relative amount of precipitate formed is Q , detailed mixing has occurred for a volume $2QV_0$. The quantity $\gamma = 2Q(w_0/w - 1)^{-1}$ is thus a measure of the proportion of the entrained fluid which has been mixed down to a molecular scale.

We have calculated γ for a number of runs, without density gradient, and the results are displayed in table 1.

Admittedly the actual processes are most complex, and so simple parameterization is likely to be impossible. (For example, once a volume of injected fluid has been diluted by two volumes of tank solution on a molecular scale, no additional dilution is detectable in our experiment.) Nevertheless, it appears that we can say that at any time something of the order of 20% of the fluid which has been entrained has been mixed down to a molecular scale. Considerations of this kind

V_0 (cm ³)	w_0 (cm/ sec)	Time (sec) ...	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
0.42	32	γ	0.14	0.18	0.21	0.20	0.19	0.18	0.16	0.15	0.14
		$(w_0/w) - 1$	2.2	3.7	5.1	6.6	8.1	9.3	11.0	12.0	13.0
		Q	0.15	0.34	0.52	0.66	0.76	0.83	0.88	0.92	0.94
0.065	9.5	γ	0.42	0.45	0.49	0.46	0.42	0.38	0.30	0.24	0.21
		$(w_0/w) - 1$	0.73	1.2	1.7	2.3	2.8	3.4	4.6	6.3	7.6
		Q	0.15	0.27	0.42	0.53	0.59	0.65	0.70	0.76	0.81
0.82	55	γ	0.32	0.35	0.35	0.32	0.28	—	—	—	—
		$(w_0/w) - 1$	2.1	3.5	4.6	5.7	6.9	—	—	—	—
		Q	0.34	0.61	0.81	0.92	0.97	—	—	—	—
0.80	9.4	γ	—	—	—	—	—	—	0.045	0.06	0.083
		$(w_0/w) - 1$	—	—	—	—	—	—	2.2	2.3	2.4
		Q	—	—	—	—	—	—	0.05	0.07	0.10
0.42	9.6	γ	—	—	—	—	—	—	0.13	—	0.17
		$(w_0/w) - 1$	—	—	—	—	—	—	1.7	—	2.1
		Q	—	—	—	—	—	—	0.11	0.14	0.18
0.81	32	γ	—	—	0.20	—	0.33	—	0.31	—	—
		$(w_0/w) - 1$	—	—	1.8	—	2.7	—	4.2	—	—
		Q	—	—	0.18	0.31	0.44	0.57	0.66	—	—
0.63	32	γ	—	0.13	0.15	0.19	0.21	—	—	—	—
		$(w_0/w) - 1$	—	1.8	3.8	4.9	6.0	—	—	—	—
		Q	—	0.12	0.29	0.46	0.62	—	—	—	—

TABLE 1. Tabulation of Q , $(w_0/w) - 1$, and $\gamma = 2Q(w_0/w - 1)^{-1}$ for small values of time in uniform fluid.

are very important when dealing with chemical reactions between mixing fluids, and in particular if concern is for possibly explosive mixtures.

A more definite, and possibly more important conclusion from our observations depends upon the fact that the presence of the density gradient had no significant effect upon Q as a function of time. This is not surprising for runs like those shown in figures 2 and 3, where Q had risen nearly to unity before the density gradient appreciably affected the motion. However, it was equally true for other runs, with lower values of $1/R_i$, for which the puff reached its maximum penetration when Q was still comparatively small.

These observations, together with the fact that the rate of horizontal spread was little affected by the density gradient, lead to the conclusion that the small-scale motions within the puff are affected little, if at all, by the presence or absence of the mean density gradient. If valid, this conclusion would run counter

to the opinion expressed by Bolgiano (1959, 1962), who argues for a large change in the nature of the small-scale motions if a density gradient exists. In our opinion the effect of the density gradient is almost entirely upon the large-scale motion. If a similar situation holds in a shear flow, the density gradient would reduce the rate of production of turbulent energy by reducing the 'mixing length'. It would, however, have little effect on the mechanisms of absorption of turbulent energy, which depend upon the small-scale motion.

At this point it is worth noting that the present measurements are fully consistent with the idea that the mixing lengths for momentum and for other properties (heat, or salt) should differ. Our puffs penetrate some distance vertically and then rebound. The effective mixing length for a property like salt is only the final resulting displacement. If the motions were superimposed upon a shear flow, however, a good deal of momentum could be exchanged near the position of maximum penetration, close to which the puff spends a considerable time before rebounding.

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